SECOND YEAR OF SURVEYING DEPARTMENT

THEORY OF ERROR 2017

LECTURE 5

STATISTICAL TESTING

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LECTURE ELEMENTS

1. "HYPOTHESIS TESTING"
2. "SYSTEMATIC DEVELOPMENT OF A TEST"
3. "TEST OF HYPOTHESIS FOR THE POPULATION MEAN"
4. "TEST OF HYPOTHESIS FOR THE POPULATION VARIANCE"
5. "TEST OF HYPOTHESIS FOR THE RATIO OF TWO POPULATION VARIANCES"

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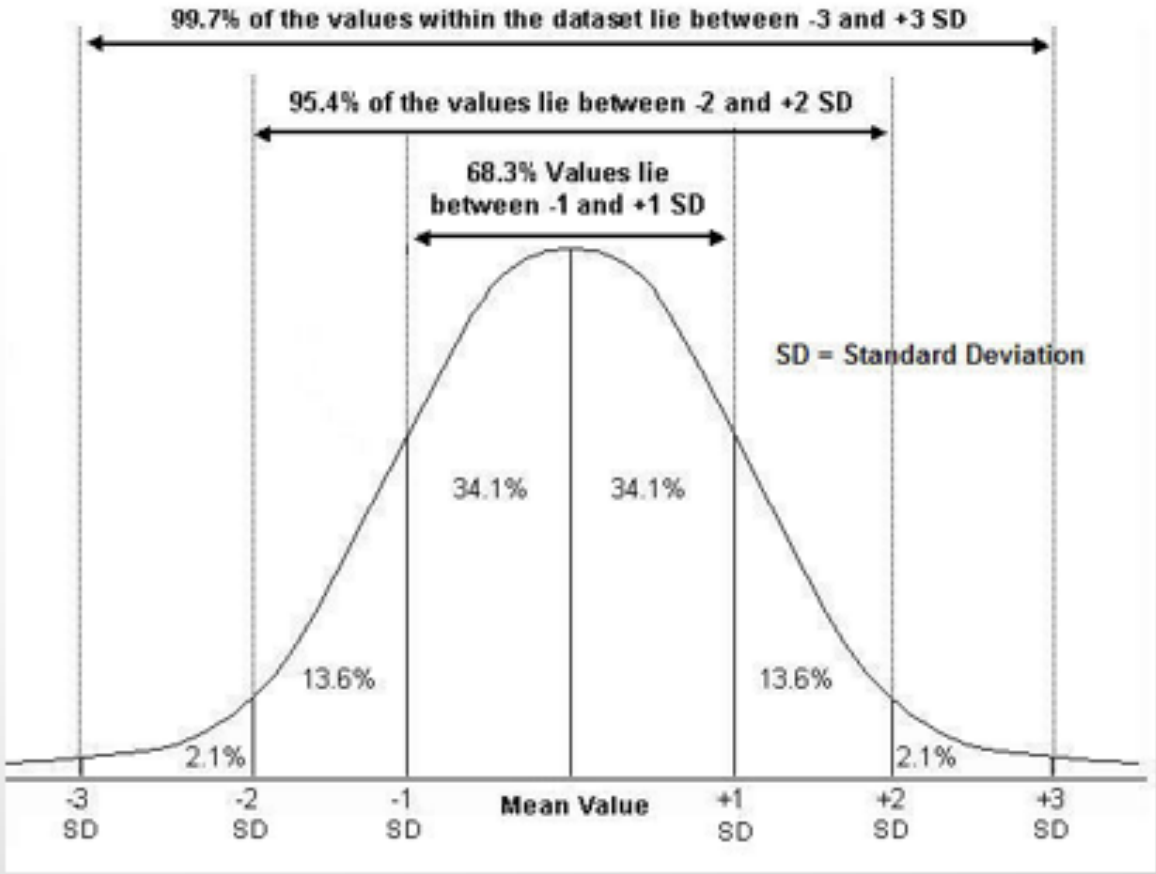
USES FOR PERCENT ERRORS

Multipliers for Various Percent Probable Errors

Symbol	Multiplier	Percent Probable Errors
E_{50}	0.6745σ	50
E_{90}	1.645σ	90
E_{95}	1.960σ	95
E_{99}	2.576σ	99
$E_{99.7}$	2.968σ	99.7
$E_{99.9}$	3.29σ	99.9



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HYPOTHESIS TESTING

- IS THE SAMPLE STATISTIC CONSISTENT WITH WHAT IS EXPECTED FROM THE POPULATION? THE PROCEDURES USED TO TEST THE VALIDITY OF A STATISTIC ARE KNOWN AS HYPOTHESIS TESTING. THE BASIC ELEMENTS OF HYPOTHESIS TESTING ARE

1. THE NULL HYPOTHESIS, H_0, IS A STATEMENT THAT COMPARES A POPULATION STATISTIC WITH A SAMPLE STATISTIC. THIS IMPLIES THAT THE SAMPLE STATISTIC IS WHAT IS "EXPECTED" FROM THE POPULATION.
2. THE ALTERNATIVE HYPOTHESIS, H_A, IS WHAT IS ACCEPTED WHEN A DECISION IS MADE TO REJECT THE NULL HYPOTHESIS, AND THUS REPRESENTS AN ALTERNATIVE POPULATION OF DATA FROM WHICH THE SAMPLE STATISTIC WAS DERIVED.

HYPOTHESIS TESTING

4. THE TEST STATISTIC IS COMPUTED FROM THE SAMPLE DATA AND IS THE VALUE USED TO DETERMINE WHETHER THE NULL HYPOTHESIS SHOULD BE REJECTED.

5. THE REJECTION REGION IS THE VALUE FOR THE TEST STATISTIC WHERE THE NULL HYPOTHESIS IS REJECTED. IN REFERENCE TO CONFIDENCE INTERVALS, THIS NUMBER TAKES THE PLACE OF THE CONFIDENCE INTERVAL BOUNDS.

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SYSTEMATIC DEVELOPMENT OF A TEST

Test Variables and Statistical Tests

Variable 1, Test Statistic	Variable 2, Sample Statistic	Null Hypothesis	Test Statistic
Population mean, μ	Sample mean, $\bar{y}$	H_0: $\mu = \bar{y}$	t
Population variance, σ^2	Sample variance, S^2	H_0: $\sigma^2 = S^2$	χ^2
Ratio of sample variances equals 1	S_1^2/S_2^2	H_0: $S_1^2/S_2^2 = 1$	F

TEST OF HYPOTHESIS FOR THE POPULATION MEAN

AT TIMES IT MAY BE DESIRABLE TO TEST A SAMPLE MEAN AGAINST A KNOWN VALUE. THE T DISTRIBUTION IS USED TO BUILD THIS TEST. THE NULL HYPOTHESIS FOR THIS TEST CAN TAKE TWO FORMS: ONE- OR TWO-TAILED TESTS. THESE TWO TESTS ARE SHOWN BELOW.

	One-Tailed Test	*Two-Tailed Test*
Null hypothesis:	H_a: $\mu = \bar{y}$	H_0: $\mu = \bar{y}$
Alternative hypothesis:	H_a: $\mu > \bar{y} (\mu < \bar{y})$	H_a: $\mu \neq \bar{y}$

THE TEST STATISTIC IS

$$t = \frac{\bar{y} - \mu}{S/\sqrt{n}}$$

IT SHOULD BE STATED THAT FOR LARGE SAMPLES (N > 30), THE *T* VALUE CAN BE REPLACED BY THE STANDARD NORMAL VALUE , Z.

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TEST OF HYPOTHESIS FOR THE POPULATION MEAN

EXAMPLE 5.2 A BASELINE OF CALIBRATED LENGTH 400.008 M IS OBSERVED REPEATEDLY WITH AN EDM INSTRUMENT. AFTER 20 OBSERVATIONS, THE AVERAGE OF THE OBSERVED DISTANCES IS 400.012 M WITH A STANDARD DEVIATION OF $\pm$0.002 M. IS THE DISTANCE OBSERVED SIGNIFICANTLY DIFFERENT FROM THE DISTANCE CALIBRATED AT A 0.05 LEVEL OF SIGNIFICANCE?

THE NULL HYPOTHESIS IS $H_0: \mu = 400.012$

AND THE ALTERNATIVE HYPOTHESIS IS $H_a: \mu \neq 400.012$

THE TEST STATISTIC IS $$t = \frac{\bar{y} - \mu}{S/\sqrt{n}} = \frac{400.012 - 400.008}{0.002/\sqrt{20}} = 8.944$$

AND THE REJECTION REGION IS $t = 8.944 > t_{\alpha/2}$

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TEST OF HYPOTHESIS FOR THE POPULATION MEAN

$$400.011 = 400.012 - 2.093\left(\frac{0.002}{\sqrt{20}}\right)$$

$$\leq \mu \leq 400.012 + 2.093\left(\frac{0.002}{\sqrt{20}}\right) = 400.013$$

NOTE THAT THE 95% CONFIDENCE INTERVAL FAILS TO CONTAIN THE BASELINE VALUE OF 400.008, AND SIMILARLY, THERE IS REASON TO BE CONCERNED ABOUT THE CALIBRATION STATUS OF THE INSTRUMENT. THAT IS, IT MAY NOT BE WORKING PROPERLY AND SHOULD BE REPAIRED.

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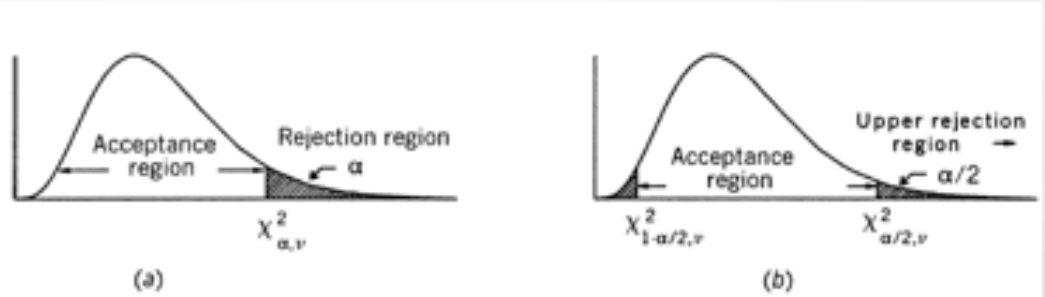
TEST OF HYPOTHESIS FOR THE POPULATION VARIANCE

THE SURVEYOR MAY ALSO WANT TO CHECK IF THE INSTRUMENT IS MEASURING AT ITS PUBLISHED PRECISION. THE X^2 DISTRIBUTION IS USED WHEN COMPARING THE VARIANCE OF A SAMPLE SET AGAINST THAT OF A POPULATION. THIS TEST INVOLVES CHECKING THE VARIANCE COMPUTED FROM A SAMPLE SET OF OBSERVATIONS AGAINST THE PUBLISHED VALUE (THE EXPECTED VARIANCE OF THE POPULATION).

	One-Tailed Test	*Two-Tailed Test*
Null hypothesis:		
	$H_0: S^2 = \sigma^2$	$H_0: S^2 = \sigma^2$
Alternative hypothesis:		
	$H_a: S^2 > \sigma^2$ (or $H_a: S^2 < \sigma^2$)	$H_a: S^2 \neq \sigma^2$

The test statistic is

$$\chi^2 = \frac{\nu S^2}{\sigma^2}$$



Graphical interpretation of (*a*) one- and (*b*) two-tailed tests.

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TEST OF HYPOTHESIS FOR THE POPULATION VARIANCE

EXAMPLE 5.3 THE OWNER OF A SURVEYING FIRM WANTS ALL SURVEYING TECHNICIANS TO BE ABLE TO READ A PARTICULAR INSTRUMENT TO WITHIN ± 1.5". TO TEST THIS VALUE, THE OWNER ASKS THE SENIOR FIELD CREW CHIEF TO PERFORM A READING TEST WITH THE INSTRUMENT. THE CREW CHIEF READS THE CIRCLE 30 TIMES AND OBTAINS ± 0.9". DOES THIS SUPPORT THE 1.5" LIMIT AT A 5% LEVEL OF SIGNIFICANCE?

THE NULL HYPOTHESIS IS $H_0: S^2 = \sigma^2$

AND THE ALTERNATIVE HYPOTHESIS IS $H_a: S^2 > \sigma^2$

THE TEST STATISTIC IS $\chi^2 = \dfrac{(30 - 1)0.9^2}{1.5^2} = 10.44$

THE NULL HYPOTHESIS IS REJECTED WHEN THE COMPUTED TEST STATISTIC EXCEEDS THE TABULATED VALUE, OR WHEN THE FOLLOWING STATEMENT IS TRUE: $\chi^2 = 10.44 > \chi^2_{\alpha,\nu} = \chi^2_{0.05,29} = 42.56$

SINCE THE COMPUTED X^2 VALUE (10.44) 0.05,29 IS LESS THAN THE TABULATED VALUE (42.56), THE NULL HYPOTHESIS CANNOT BE REJECTED.

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TEST OF HYPOTHESIS FOR THE RATIO OF TWO POPULATION VARIANCES

AFTER DOING BOTH ADJUSTMENTS, THE POST-ADJUSTMENT REFERENCE VARIANCES CAN BE COMPARED. IF THE CONTROL IS WITHOUT ERROR AND NO SYSTEMATIC ERRORS ARE PRESENT IN THE DATA, THE RATIO OF THE TWO REFERENCE VARIANCES SHOULD BE CLOSE TO 1.

	One-Tailed Test	*Two-Tailed Test*
Null hypothesis:	$H_0: \frac{S_1^2}{S_2^2} = 1 \quad (\text{i.e., } S_1^2 = S_2^2)$	$H_0: \frac{S_1^2}{S_2^2} = 1 \quad (\text{i.e., } S_1^2 = S_2^2)$
Alternative hypothesis:	$H_a: \frac{S_1^2}{S_2^2} > 1 \quad (\text{i.e., } S_1^2 > S_2^2)$	$H_a: \frac{S_1^2}{S_2^2} \neq 1 \quad (\text{i.e., } S_1^2 \neq S_2^2)$

THE TEST STATISTIC THAT WILL BE USED TO DETERMINE REJECTION OF THE NULL HYPOTHESIS IS

$$F = \frac{S_1^2}{S_2^2} \quad \text{or} \quad F = \frac{S_2^2}{S_1^2} \qquad F = \frac{\text{larger sample variance}}{\text{smaller sample variance}}$$

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TEST OF HYPOTHESIS FOR THE POPULATION VARIANCE

EXAMPLE 5.6 A BASELINE IS OBSERVED REPEATEDLY OVER A PERIOD OF TIME USING AN EDM INSTRUMENT. EACH DAY, 10 OBSERVATIONS ARE TAKEN AND AVERAGED. THE VARIANCES FOR THE OBSERVATIONS ARE LISTED BELOW. AT A SIGNIFICANCE LEVEL OF 0.05, ARE THE RESULTS OF DAY 2 SIGNIFICANTLY DIFFERENT FROM THOSE OF DAY 5?

THE NULL HYPOTHESIS IS $H_0: \frac{S_2^2}{S_5^2} = 1$

AND THE ALTERNATIVE HYPOTHESIS IS $H_a: \frac{S_2^2}{S_5^2} \neq 1$

THE TEST STATISTIC IS $F = \frac{61}{54} = 1.13$

IN THIS CASE THE REJECTION REGION IS $F = 1.13 > F_{0.025,9,9} = 4.03$ AND IS NOT SATISFIED. CONSEQUENTLY, THE TEST FAILS TO REJECT THE NULL HYPOTHESIS, AND THERE IS NO STATISTICAL REASON TO BELIEVE THAT THE DATA OF DAY 2 ARE STATISTICALLY DIFFERENT FROM THOSE OF DAY 5.

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THANKS

PLEASE DON'T USE THIS PRESENTATION WITHOUT GETTING A PERMEATION FROM ITS ORIGINAL OWNER

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